Chapter (1)

Units and Dimensions

Example (1.1)

Show that the following equation is dimensionally correct? $X = \upsilon_o t + \frac{1}{2} a t^2$

$$LHS = [L]$$

$RHS = [LT^{-1}] * [T] + [LT^{-2}] * [T]^2 = [L]$ $RHS = LHS$ العلاقة صحيحة

Example (1.2)

Suppose you wish to derive a formula for the distance x traveled by a car in a time of t if the car starts from rest and moves with constant acceleration a. Check the validity of this expression $x = \frac{1}{2} a t^2$.

$$LHS = [L]$$

$$RHS = [LT^{-2}] * [T]^2 = [L]$$

The equation is dimensionally correct.

Example (1.3)

Show that the expression $\upsilon = \upsilon_o + a\, t$ is dimensionally correct ?

$$LHS = [LT^{-1}]$$

$$RHS = [LT^{-1}] + [LT^{-2}][T] = [LT^{-1}]$$

The equation is dimensionally correct

Example (1.4)

Suppose we are told that the acceleration a of a particle moving in a circle of radius r with uniform velocity υ is proportional to some power of r, say r^n, and some power of υ, say υ^m. How can we determine the powers of r and υ ?

$$a = k\, r^n \upsilon^m$$

$$LHS = [LT^{-2}]$$

$$[L][T^{-2}] = [L]^n [LT^{-1}]^m = [L]^{n+m} [T]^{-m}$$

For [T]; $m = 2$

For [L]; $n + m = 1$ $n = -1$

Example (1.5)

The period (T) of a simple pendulum is the time for one complete swing. How does (T) depends on the mass (m) of the bob, the length (L) of the string and gravitational (g)?

$$T = (\text{constant})\ m^a L^b g^c$$

$$[M]^0[L]^0[T] = [M]^a [L]^b [LT^{-2}]^c = [M]^a [L]^{b+c} [T]^{-2c}$$

For [M]; $a = 0$

For [T]; $1 = -2c$ $c = -1/2$

For [L]; $0 = b + c$ $b = 1/2$

$$T = (\text{constant})\ L^{1/2} g^{-1/2} = (\text{constant}) \sqrt{\frac{L}{g}}$$

Example(1.9)

The speed of a particle varies in time according to $\upsilon = A\,t - B\,t^3$. What are the dimensions of A and B?

$\upsilon = A\,t - B\,t^3$ (معلوم أن العلاقة صحيحة ولذلك فإن الأبعاد هى نفسها فى الطرفين)

$[LT^{-1}] = [A][T] - [B][T]^3$

$[L][T^{-1}] = [A][T] - [B][T]^3$

So, $[L][T^{-1}] = [A][T]$, $[A] = [LT^{-2}]$

And, $[L][T^{-1}] = [B][T]^3$, $[B] = [LT^{-4}]$

Example(1.10)

According to Newton's second law of motion, the force F acting on a particle is related to its mass m and acceleration a, according to F = m a. According to Newton's law of gravitation there is an attractive force between particles given by: $F = G\,m_1\,m_2 / r^2$, where r is the distance between the particles. What are the dimensions of G ?

$F = G\,m_1\,m_2 / r^2$

$[M\,L\,T^{-2}] = [G]\,[M]\,[M] / [L^2] = [G]\,[M^2]\,[L^{-2}]$

$[G] = [L]\,[M^{-1}]\,[T^{-2}]$

Also; The unit of G is $m^3/Kg.s^2$

Chapter (2)

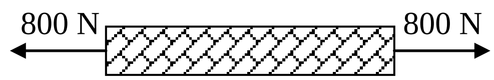
Elasticity

Mechanical Properties of Metals

Example (2.1)

The bar shown has a square cross section(مقطع مربع) for which the depth and thickness are 40 mm. If an axial force of 800 N is applied along the centroidal axis of the bar's cross sectional area, determine the average normal stress acting on the bar?(عين متوسط الاجهاد الواقع الشريط)

Answer



Stress is $\sigma = \frac{F}{A} = \frac{800}{(40*10^{-3})^2} = 500*10^3$ **Pa**

Example (2.2)

A 80 Kg mass is hung (علقت) on a steel wire having 18m long and 3mm diameter. What is the elongation of the wire, knowing Young's modulus for steel is 21 x 10^{10} N/m^2? **كتلة ٨٠ كج علقت فى سلك طوله ١٨ متر و قطره ٣مم مامقدار التمدد فى طوله**

Answer

$m = 80\ kg$ $\quad L_o = 18\ m$

$2r = 3\ mm = 0.003m$ $\quad r = 0.0015m$ $\quad Y = 21 \times 10^{10}\ N/m^2$

Young's modulus is given by $Y = \frac{F/A}{\Delta L/L_o}$ *so the elongation is* $\Delta L = \frac{F}{A}\frac{L_o}{Y}$

$$\boxed{\Delta L = \frac{80x9.8}{\pi(0.0015)^2}x\frac{18}{21x10^{10}} = 0.0095m = 9.5mm}$$

Exemple (2.3)

A piece of copper originally 305 mm long is pulled in tension with a stress of 276 MPa. If the deformation is entirely elastic, what will be the resultant elongation?

Answer

Young's modulus is given by $Y = \frac{F/A}{\Delta L/L_o}$ then

$$\Delta L = \frac{(F/A)L}{Y} = \frac{\sigma L}{Y} = \frac{(276x10^{6})(305x10^{-3})}{(11x10^{11})} = 0.76x10^{-3}m$$

Example (2.4)

A person carries a 21 kg suitcase in one hand. Assuming the humerus bone (عضد العظم)supports the entire weight(وزن), determine how much it stretches(الاستطالة). Assume the humerus is 33 cm in length and has an effective cross-sectional area of 5.2 x 10^{-4} m^2. where $Y=1.6x10^{10}$ N/m^2

Answer

Young's modulus is

$$Y = \frac{F/A}{\Delta L/L} \Rightarrow \Delta L = \frac{FL}{YA} = \frac{mgL}{YA} = \frac{(21)(9.8)(0.33)}{(1.6x10^{10})(5.2x10^{-4})} = 8.17x10^{-6}m$$

Exemple (2.5)

A telephone wire 120 m long and 2.2 mm in diameter is stretched by a force of 380 N. What is the longitudinal stress? If the length after stretching is 120.10 m, what is the longitudinal strain? Determine Young's modulus for the wire?

Answer

$$A = \pi r^2 = 3.14 * (1.1*10^{-3})^2 = 3.8 * 10^{-6}\ m^2$$

$$\sigma = \frac{F}{A} = \frac{380}{3.8*10^{-6}} = 100*10^{6}\ N/m^2 = 100\ MPa$$

$$\Delta L = 120.1 - 120 = 0.1\ m$$

$$\varepsilon = \frac{\Delta L}{L_o} = \frac{0.1}{120} = 8.33*10^{-4}$$

$$Y = \frac{\sigma}{\varepsilon} = \frac{100*10^{6}}{8.33*10^{-4}} = 12*10^{10}\ N/m^2 = 12*10^{4}\ MPa$$

Exemple (2.7)

A load of 102 kg is supported by a wire of length 2 m and cross sectional area 0.1 cm^2. The wire is stretched by 0.22 cm . Find the tensile stress, tensile strain, and Young's modulus of the wire ?

m = 102 kg L = 2 m A = 0.1 cm^2 ΔL = 0.22 cm

$$\text{Tensile Stress}(\sigma) = \frac{F}{A} = \frac{mg}{A} = \frac{102*9.8}{0.1*10^{-4}} = 999.6*10^{5}\ N/m^2$$

$$\text{Tensile Strain } (\varepsilon) = \frac{\Delta L}{L_o} = \frac{0.22}{2*100} = 11*10^{-4}$$

$$\text{Young's modulus (Y)} = \frac{\sigma}{\varepsilon} = \frac{999.6 * 10^5}{11 * 10^{-4}} = 90.87 * 10^9 \ \text{N/m}^2$$

Exemple (2.8)

A structure steel rod has a radius R of 9.5 mm and a length L of 81 cm. A force F of $6.2 * 10^4$ N stretches it axially. ($E_{steel} = 2 * 10^{11}$ N/m^2)

(a) What is the stress in the rod ?

(b) What is the elongation of the rod under this load ?

(c) What is the strain?

Radius = 9.5 mm L = 81 cm F = $6.2 * 10^4$ N

$$\text{Tensile Stress} (\sigma) = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{6.2*10^4}{\pi (9.5*10^{-3})^2} = 2.19*10^8 \ \text{N/m}^2$$

$$\text{Tensile Strain } (\varepsilon) = \frac{\Delta L}{L_o} \quad ; \quad E = \frac{\sigma}{\varepsilon}$$

$$\Delta L = \varepsilon * L_o = \frac{\sigma}{Y} * L_o = \frac{2.19 * 10^8}{2 * 10^{11}} * 81 * 10^{-2} = 8.86 * 10^{-4} \ \text{m}$$

$$\text{Tensile Strain } (\varepsilon) = \frac{\sigma}{Y} = \frac{2.19 * 10^8}{2 * 10^{11}} = 1.1 * 10^{-3}$$

Example (2.9)

A horizontal force(قوة افقية) of 1.2 N is applied to the top of a stack of pancakes (كومة من الفطائر)13 cm in diameter(قطر) and 9 cm high(ارتفاع). The result is a 2.5 cm shear. Find the shear modulus.

Answer

$$\text{Shear Modulus (S)} = (\mathbf{F} / \mathbf{A}) / \theta = (\mathbf{F} / \mathbf{A}) / (\Delta x / h)$$

$$S = \frac{F \ h}{A \ \Delta x} = \frac{F \ h}{\pi r^2 \ \Delta x} = \frac{(1.2)(0.09)}{\pi (0.13/2)^2 (0.025)} \qquad \text{N/m}^2$$

Exemple (2.12)

A wire of length 120cm and diameter 0.82mm, supported from one end, A 5.3kg in the other end . Find :

a) The stress in the wire

b) The strain in the wire

c) The strain energy where Y = 1.2x 10^{12} dyne/cm^2 and g = 980cm/sec^2

Solution

$r = \frac{0.82}{20} = 0.041\,cm$ and $m = 5.3x10^3 gm$

The stress $= \frac{F}{A} = \frac{m\,g}{A} = \frac{(5.3x10^3)980}{\pi(0.041)^2} =$ dyne /cm^2

The strain $= \frac{Stress}{Y} =$

Strain Energy $= \frac{1}{2}(Stress)(Strain) =$

Exemple (2.13)

A uniform wire of length 20cm density 0.78gm / cm^3 and mass 16gm stretched by a distance 1.2mm when 8kg is supported on it , Find :

a) The stress in the wire , b) Young's modulus, c) The strain energy

Solution

Volume $V = \frac{m}{\rho} = \frac{16}{7.8} = 2.05\,cm^3$

But V=A . L $\Rightarrow$ $A = \frac{V}{L} = \frac{2.05}{20} = 0.1\,cm^2$

Stress $= \frac{F}{A} = \frac{m\,g}{A} = \frac{(8x10^3)980}{0.1} =$ dyne / cm^2

But Stress = $Y\,\frac{\Delta L}{L}$ $\Rightarrow$ $Y = \frac{Stress\,.\,20}{1.2x10^{-1}} =$ dyne / cm^2

Chapter (3)

Fluid Mechanics

Example (3.5)

In a car lift used in a service station, compressed air exerts a force on a small piston(مكبس) that has a circular cross section and a radius of 5.00cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0cm. What force must the compressed air exert to lift a car weighing 13,300N? What air pressure produces this force?

Solution

Using the Pascal's principle,

$$F_1 = \frac{A_1}{A_2} F_2 = \frac{\pi (0.05)^2}{\pi (0.15)^2}(1.33x\,10^4) = 1.48x\,10^3 N$$

The necessary pressure of the compressed air is

$$P_1 = \frac{F_1}{A_1} = \frac{1.48x\,10^3}{\pi (0.05)^2} = 1.88x\,10^5 N\ /\ m^2$$

Example (3.6)

What fraction of the total volume of an iceberg is exposed? The density of ice is ϱ_{ice} = 0.92 gm/cm^3 and that of sea water is ϱ_{water} = 1.03 gm/cm^3.

إحسب الجزء المعرض للهواء من الثلج الطافى على مياة المحيط

Solution

Weight of ice is $W_{ice} = \rho_{ice} V_{ice}\, g$ *the buoyant force of water is* $B = \rho_{water} V_{water}\, g$

$$\rho_{water} V_{water}\, g = \rho_{ice} V_{ice}\, g \quad \Rightarrow \rho_{water} V_{water} = \rho_{ice} V_{ice}$$

$$V_{water} / V_{ice} = \rho_{ice} / \rho_{water} = 0.92/1.03 = 89\%$$

The volume of ice exposed in air is 100%-89% = 11%

=============end 1st midterm=======================